Freshman WOCOMAL 2023-2024 Meet 1

October 2023

Freshman Meet 1 - October 2023

Round 1: Order of Operations

1. (Shepherd Hill Regional) Evaluate

$$3 + 5(6 + 2) + 9 - 4 \div 2$$

2. (Holy Name C.C.H.S) Evaluate

$$-x^2 + x^3 - x^4 \div x$$

for $x = -2$.

3. (Shrewsbury) Let $x \bowtie y = \frac{2x^2}{y}$ and $x \oplus y = xy - 2$ where the operation $\bowtie$ precedes operation $\oplus$. Evaluate

$$3 \bowtie 8 \oplus (-2) \bowtie (-4)$$

1. _____

2. _____

3. _____

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Round 2: Linear Equations

1. (Notre Dame) Solve for f:

$$3(f+7) = 2f$$

2. (Doherty) Solve for y:

$$\frac{1}{10}(10y-3) - \frac{3}{5}(y+1) = \frac{1}{10}$$

Express your answer as a simplified fraction.

3. (Worcester Academy) Suppose we have

$$\frac{1}{3}x - \frac{1}{5}y = t$$
$$\frac{1}{4}x + \frac{1}{2}y = t$$

Express $2x+3y$ in terms of t. Express all coefficients and constants as simplified fractions.

1. _____

2. _____

3. _____

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Round 3: Logic Problems

1. The product of two one-digit numbers is a two-digit number that ends in 5 and is larger than 37. What is the sum of these two one-digit numbers?

2. A room contains 10 flies at 12:00 PM and the number of flies in the room doubles every minute. The room becomes full at 1:00 PM. At what time was the room one-eighth of the way full?

3. Four of the faces of a cube with side length 8 are painted such that there exists a painted face that is opposite an unpainted face. The cube is then cut into cubes of side length 1. How many of these cubes of side length 1 have paint on exactly 1 of their faces?

1. _____

2. _____

3. _____

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Round 4: Ratio, Proportion, and Variation

1. On a map, 0.75 inches represents 4 miles. How many miles does 9 inches represent?

2. (Bartlett) Three numbers a, b, c are in ratio $a : b : c = 1 : 4 : 9$. Additionally, they satisfy

$$7a + 2b - 3c = 96$$

Compute $a + b + c$.

3. Suppose for $1 \leq n \leq 11$, the ratio of the number of students in the nth grade to the number of students in the $(n+1)$th grade is $\frac{n^2-1}{n}$. What is the ratio of the number of students in the 4th grade to the number of students in the 9th grade?

1. _____

2. _____

3. _____

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Team Round

1. How many ways can you choose an outfit of one shirt, one pair of pants, and one hat if you have 3 shirts, 4 pairs of pants, and 6 hats, but there is one specific shirt and one specific pair of pants that you refuse to wear at the same time?

2. (Bartlett) Solve for x:

$$2(7x - 4) - 4(2x - 6) = 3x + 31$$

3. (Bancroft) Chris has 3 sisters and 5 brothers. His sister Amy has S sisters and B brothers. What is the product of S and B?

4. A track is shaped like a right triangle with shortest sides of length 9 and 40. Suppose Alice and Bob start at the same point and walk in opposite directions. Alice walks twice as fast as Bob. What is the distance Alice walks until Alice meets Bob for the first time?

5. What is the sum of all integers from 1 to 20, inclusive, that are not divisible by 3?

6. What two-digit number is equal to twice the product of its digits?

7. If $f(x) = 35-x$ and $g(x) = 2^x$, for what value of x does $f(g(f(x))) = 3$?

8. (Houshton) Compute

$$(6 \div 2 + 1)^2 - 2[6 \div (2 + 1)]^2 + 2[(6 \div 2) + 1]^2$$

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Team Round Answer Sheet

1. _____

2. _____

3. _____

4. _____

5. _____

6. _____

7. _____

8. _____

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Round 1: Order of Operations

1. (Shepherd Hill Regional) Evaluate

$$3 + 5(6 + 2) + 9 - 4 \div 2$$

Solution: We have

$$\begin{aligned} & 3 + 5(6 + 2) + 9 - 4 \div 2 \\ &= 3 + 5 \cdot 8 + 9 - 4 \div 2 \\ &= 3 + 40 + 9 - 2 \\ &= 43 + 9 - 2 \\ &= 52 - 2 \\ &= \boxed{50} \end{aligned}$$

2. (Holy Name C.C.H.S) Evaluate

$$-x^2 + x^3 - x^4 \div x$$

for $x = -2$.

Solution: We have

$$\begin{aligned} & -x^2 + x^3 - x^4 \div x \\ &= -(-2)^2 + (-2)^3 - (-2)^4 \div (-2) \\ &= -(-2)^2 + (-2)^3 - (-2)^3 \\ &= -4 - 8 - (-8) \\ &= -12 - (-8) \\ &= \boxed{-4} \end{aligned}$$

3. (Shrewsbury) Let $x \bowtie y = \frac{2x^2}{y}$ and $x \oplus y = xy - 2$ where the operation $\bowtie$ precedes operation $\oplus$. Evaluate

$$3 \bowtie 8 \oplus (-2) \bowtie (-4)$$

Solution: We have

$$\begin{aligned} & 3 \bowtie 8 \oplus (-2) \bowtie (-4) \\ &= \frac{2(3)^2}{8} \oplus \frac{2(-2)^2}{(-4)} \\ &= \frac{9}{4} \oplus (-2) \\ &= \frac{9}{4} \cdot (-2) - 2 \\ &= -\frac{9}{2} - 2 \\ &= \boxed{-\frac{13}{2}} \end{aligned}$$

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Round 2: Linear Equations

1. (Notre Dame) Solve for f:

$$3(f+7) = 2f$$

Solution: We have

$$\begin{aligned} 3(f+7) &= 2f \\ \Rightarrow 3f + 21 &= 2f \\ \Rightarrow f &= \boxed{-21} \end{aligned}$$

2. (Doherty) Solve for y:

$$\frac{1}{10}(10y-3) - \frac{3}{5}(y+1) = \frac{1}{10}$$

Express your answer as a simplified fraction.

Solution: Solution 1: We have

$$\begin{aligned} \frac{1}{10}(10y-3) - \frac{3}{5}(y+1) &= \frac{1}{10} \\ \Rightarrow y - \frac{3}{10} - \frac{3}{5}y - \frac{3}{5} &= \frac{1}{10} \\ \Rightarrow \frac{2}{5}y &= 1 \\ \Rightarrow y &= \boxed{\frac{5}{2}} \end{aligned}$$

Solution 2: We have

$$\begin{aligned} \frac{1}{10}(10y-3) - \frac{3}{5}(y+1) &= \frac{1}{10} \\ \Rightarrow (10y-3) - 6(y+1) &= 1 \\ \Rightarrow 10y - 3 - 6y - 6 &= 1 \end{aligned}$$

$$\Rightarrow 4y = 10$$
$$\Rightarrow y = \boxed{\frac{5}{2}}$$

3. (Worcester Academy) Suppose we have
$$\frac{1}{3}x - \frac{1}{5}y = t$$
$$\frac{1}{4}x + \frac{1}{2}y = t$$
Express $2x+3y$ in terms of t. Express all coefficients and constants as simplified fractions.

Solution: We have
$$\frac{1}{3}x - \frac{1}{5}y = t$$
$$\Rightarrow \frac{5}{3}x - y = 5t \tag{1}$$
Also,
$$\frac{1}{4}x + \frac{1}{2}y = t$$
$$\Rightarrow \frac{1}{2}x + y = 2t \tag{2}$$
Adding (1) and (2), we get
$$(\frac{5}{3} + \frac{1}{2})x = 7t$$
$$\Rightarrow \frac{13}{6}x = 7t$$
$$\Rightarrow x = \frac{42}{13}t$$
Plugging this into (2), we get
$$\frac{1}{2}\frac{42}{13}t + y = 2t$$

$$\Rightarrow \frac{21}{13}t + y = 2t$$

$$\Rightarrow y = \frac{5}{13}t$$

Thus,

$$2x + 3y$$

$$= 2(\frac{42}{13}t) + 3(\frac{5}{13}t)$$

$$= \frac{84}{13}t + \frac{15}{13}t$$

$$= \boxed{\frac{99}{13}t}$$

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Round 3: Logic Problems

1. The product of two one-digit numbers is a two-digit number that ends in 5 and is larger than 37. What is the sum of these two one-digit numbers?

Solution: Since the product ends in 5, one of the one-digit numbers must be 5. We see that 45 is the only number that is larger than 37 and is the product of 5 and another one-digit number. Thus, the other one-digit number is 9. The sum of 5 and 9 is $\boxed{14}$.

2. A room contains 10 flies at 12:00 PM and the number of flies in the room doubles every minute. The room becomes full at 1:00 PM. At what time was the room one-eighth of the way full?

Solution: The fact that the room starts with 10 flies is irrelevant. If the room becomes full at 1:00 PM, the room was half full at 12:59 PM. Then, the room was a quarter full at 12:58 PM. Finally, we get that the room was an eighth of the way full at $\boxed{12:57}$ PM.

3. Four of the faces of a cube with side length 8 are painted such that there exists a painted face that is opposite an unpainted face. The cube is then cut into cubes of side length 1. How many of these cubes of side length 1 have paint on exactly 1 of their faces?

Solution: There is only 1 possible configuration of these four faces up to rotation. Two of these painted faces are each adjacent to two other painted faces, and the other two of the painted faces are each adjacent to 3 other painted faces. Cubes of side length 1 on the edge between two painted faces are painted on more than 1 side. Thus, on the painted faces that are adjacent

to two other painted faces, there is a 7 by 7 square of cubes of side length 1 with only one face painted. On the painted faces that are adjacent to three other painted faces, there is a 6 by 7 rectangle of cubes of side length 1 with only one face painted. Thus, there is a total of $2 \cdot 7 \cdot 7 + 2 \cdot 6 \cdot 7 = \boxed{182}$ cubes of side length 1 with exactly one face painted.

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Round 4: Ratio, Proportion, and Variation

1. On a map, 0.75 inches represents 4 miles. How many miles does 9 inches represent?

Solution: The answer is

$$9 \text{ inches} \cdot \frac{4 \text{ miles}}{0.75 \text{ inches}} = 9 \cdot \frac{4}{\frac{3}{4}} \text{ miles}$$

$$= \boxed{48} \text{ miles}$$

2. (Bartlett) Three numbers a, b, c are in ratio $a : b : c = 1 : 4 : 9$. Additionally, they satisfy

$$7a + 2b - 3c = 96$$

Compute $a + b + c$.

Solution: We have $b = 4a$ and $c = 9a$. Thus,

$$7a + 2b - 3c = 96$$

$$\Rightarrow 7a + 2(4a) - 3(9a) = 96$$

$$\Rightarrow 7a + 8a - 27a = 96$$

$$\Rightarrow -12a = 96$$

$$\Rightarrow a = -8$$

$$\Rightarrow a + b + c = a + 4a + 9a = 14a = 14(-8) = \boxed{-112}$$

3. Suppose for $1 \leq n \leq 11$, the ratio of the number of students in the nth grade to the number of students in the $(n+1)$th grade is $\frac{n^2-1}{n}$. What is the ratio of the number of students in the 4th grade to the number of students in the 9th grade?

Solution: We have that the ratio of the number of students in the nth grade to the number of students in the $(n+1)$th grade is

$$\frac{n^2-1}{n} = \frac{(n+1)(n-1)}{n}$$

Thus, the ratio we are looking for is

$$= (\frac{5 \cdot 3}{4})(\frac{6 \cdot 4}{5})(\frac{7 \cdot 5}{6})(\frac{8 \cdot 6}{7})(\frac{9 \cdot 7}{8})$$
$$= 3 \cdot 5 \cdot 6 \cdot 7 \cdot 9 = \boxed{5670}$$

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Team Round

1. How many ways can you choose an outfit of one shirt, one pair of pants, and one hat if you have 3 shirts, 4 pairs of pants, and 6 hats, but there is one specific shirt and one specific pair of pants that you refuse to wear at the same time?

Solution: There are $3 \cdot 4 \cdot 6 = 72$ ways to choose one shirt, one air of pants, and one hat. We have that 6 of those combinations have the specific shirt and specific pair of pants that cannot be worn at the same time. Thus, there are $72 - 6 = \boxed{66}$ ways to choose an outfit.

2. (Bartlett) Solve for x:

$$2(7x - 4) - 4(2x - 6) = 3x + 31$$

Solution: We have

$$2(7x - 4) - 4(2x - 6) = 3x + 31$$
$$\Rightarrow 14x - 8 - 8x + 24 = 3x + 31$$
$$\Rightarrow 3x = 15$$
$$\Rightarrow x = \boxed{5}$$

3. (Bancroft) Chris has 3 sisters and 5 brothers. His sister Amy has S sisters and B brothers. What is the product of S and B?

Solution: There are a total of 3 sisters and 6 brothers. Then, Amy has 2 sisters and 6 brothers. The product is $2 \cdot 6 = \boxed{12}$.

4. A track is shaped like a right triangle with shortest sides of length 9 and 40. Suppose Alice and Bob start at the same point and walk

in opposite directions. Alice walks twice as fast as Bob. What is the distance Alice walks until Alice meets Bob for the first time?

Solution: Using the Pythagorean Theorem, the third side length is $\sqrt{9^2+40^2} = 41$. Thus, the total length of the track is $9+40+41 = 90$. Since Alice walks twice as fast, she will cover a two thirds of the track while Bob will cover a one third of the track. Thus, Alice walks a distance of $\frac{2}{3} \cdot 90 = \boxed{60}$.

5. What is the sum of all integers from 1 to 20, inclusive, that are not divisible by 3?

Solution: The sum of the integers from 1 to 20 is $\frac{20 \cdot 21}{2} = 210$. The sum of the multiples of 3 less than 20 is $\frac{3+18}{2} \cdot 6 = 63$. Thus, the sum of the integers from 1 to 20 not divisible by 3 is $210 - 63 = \boxed{147}$.

6. What two-digit number is equal to twice the product of its digits?

Solution: We have

$$10a + b = 2ab$$
$$\Rightarrow 2ab - 10a - b + 5 = 5$$
$$\Rightarrow (2a-1)(b-5) = 5$$

Since both factors are integers, either $(2a-1, b-5) = (1,5)$ or $(2a-1, b-5) = (5,1)$. The former gives solution $(a,b) = (1,10)$, which is not possible. The latter then gives the only solution $(a,b) = (3,6)$, which gives the two-digit number, $\boxed{36}$.

7. If $f(x) = 35-x$ and $g(x) = 2^x$, for what value of x does $f(g(f(x))) = 3$?

Solution: We have that

$$35 - 2^{35-x} = 3$$

$$\Rightarrow 2^{35-x} = 32$$
$$\Rightarrow 35 - x = 5$$
$$\Rightarrow x = \boxed{30}$$

8. (Houshton) Compute

$$(6 \div 2 + 1)^2 - 2[6 \div (2 + 1)]^2 + 2[(6 \div 2) + 1]^2$$

Solution:

$$(6 \div 2 + 1)^2 - 2[6 \div (2 + 1)]^2 + 2[(6 \div 2) + 1]^2$$
$$= (3 + 1)^2 - 2[6 \div 3]^2 + 2[3 + 1]^2$$
$$= 16 - 2 \cdot 4 + 2 \cdot 16$$
$$= 16 - 8 + 32 = \boxed{40}$$